Monthly Problems 1

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Problems

1. A mouse starts on the bottom-left $(1,1)$ square of a $26^{26} \times 26^{26}$ grid. Each second, the mouse is allowed to move to the square to its right or above it. George is allowed to put fences between any two squares $(1,i)$ and $(2,i)$ and between $(j,1)$ and $(j,2)$ $\forall i,j \in \{1,2,\ldots,26^{26}\}$, so that the mouse cannot move between those two squares. Without any fences, the mouse can reach the top-right square $(26^{26},26^{26})$ in N distinct paths. Find the number of ways that George can place fences so that the mouse can reach the top-right square in exactly $\frac{N}{2}$ distinct paths. *(Michael Ma)*

2. Does there exist positive integers n such that $26^{n^{26}}$ could be expressed as the sum of consecutive cubes? *(George Zhu)*

3. Let M be the mid-point of minor arc BC of the circumcirle of acute $\triangle ABC$ with circuradius R. AM intersects BC at X. The perpendicular from A intersects BC and the circumcircle of ABC at D and E respectively. The perpendicular from X intersects AE at F. Let R_1 be the circumradius of $\triangle AXC$. Prove that

$$\frac{EM}{XF} = \frac{R}{R_1}$$

(George Zhu)

4. $a_i (1 \leq i \leq n)$ are positive reals such that their product equals to 1. Prove that

$$\prod_{i=1}^{n} \left(4a_i^n + (n+3)^2\right) \geq (8n^n + 12^n)^n$$

(George Zhu)